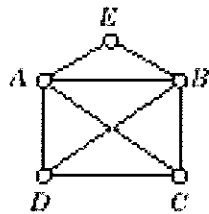


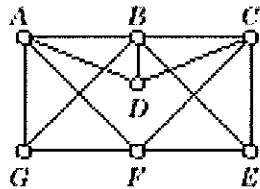
## 4.4 Euler Circuits and Paths

Euler Circuit:

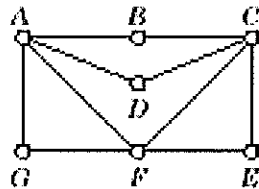
Euler Path:



Graph 1



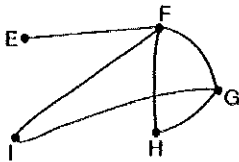
Graph 2



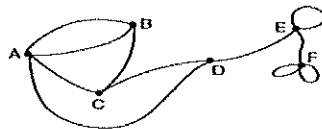
Graph 3

Do the following graphs have an Euler circuit? A path? If so, name it.

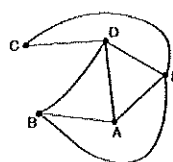
1.



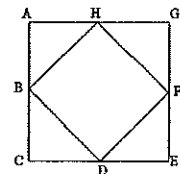
2.



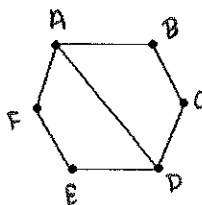
3.



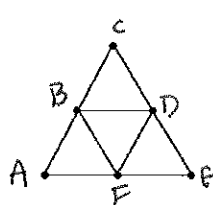
4.



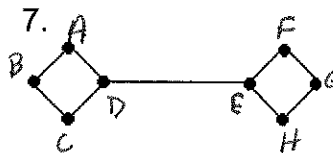
5.



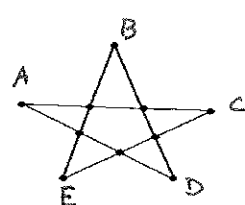
6.



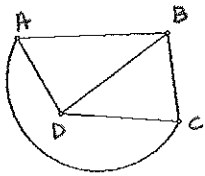
7.



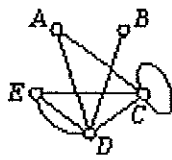
8.



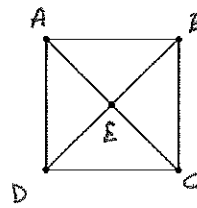
9.



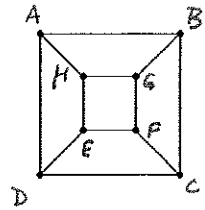
10.



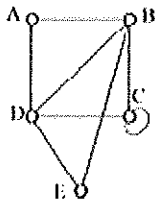
11.



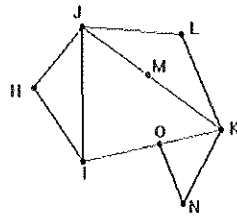
12.



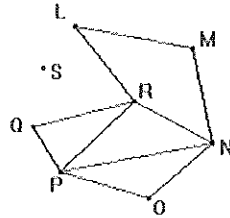
13.



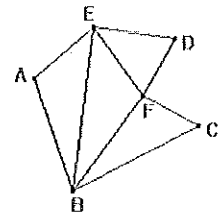
14.



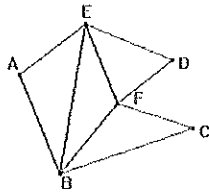
15.



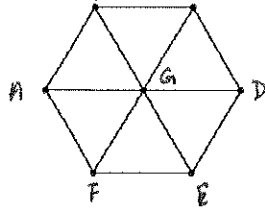
16.



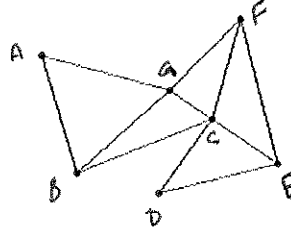
17.



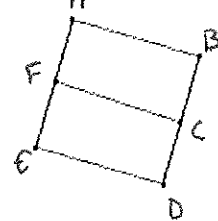
18.



19.



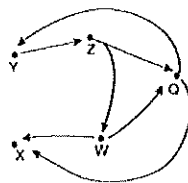
20.



### Digraph

Indegree:

Outdegree:



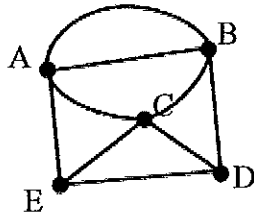
## 4.5 – Hamiltonian Circuits and Paths

### Hamiltonian Path

### Hamiltonian Circuit

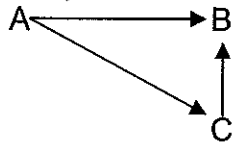
If a connected graph has  $n$  vertices ( $n > 2$ ) and each vertex has degree at least  $\frac{n}{2}$ , then the graph has a Hamiltonian circuit.

Example:



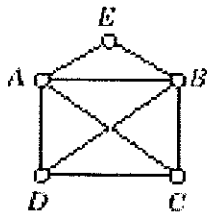
A tournament is a digraph that results from giving directions to the edges of a complete graph.

Example:

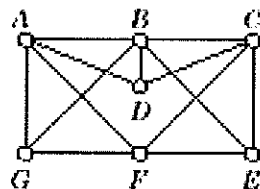


This tournament illustrates that A beats B, A beats C, and C beats B.

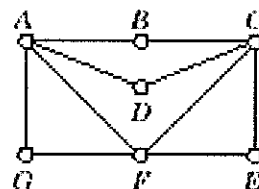
Every tournament has at least one Hamiltonian path. If there is exactly one Hamiltonian path, it can be used to rank teams in order, from winner to loser.



Graph 1



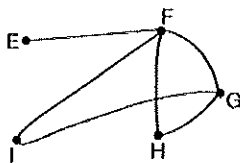
Graph 2



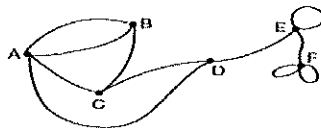
Graph 3

Do the following graphs have a Hamiltonian circuit? A path? If so, name it.

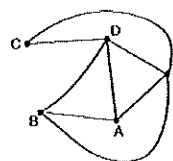
1.



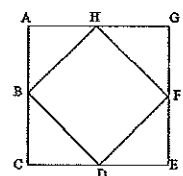
2.

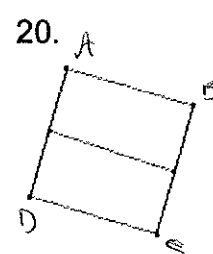
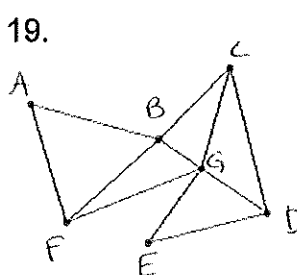
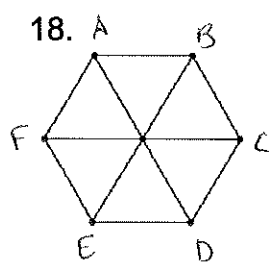
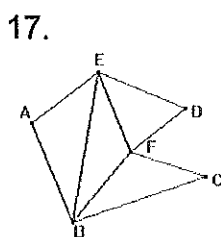
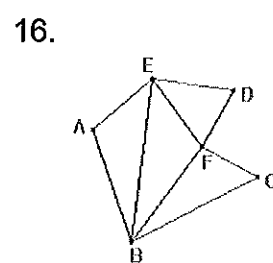
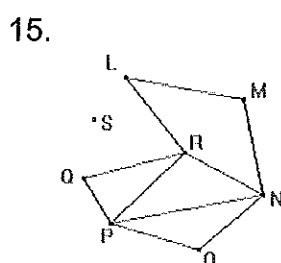
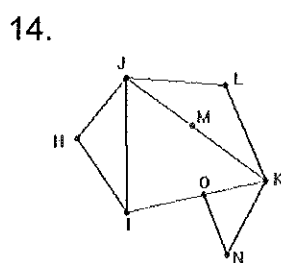
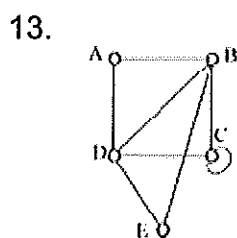
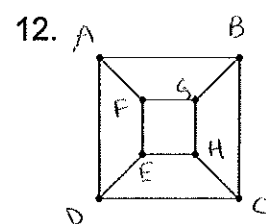
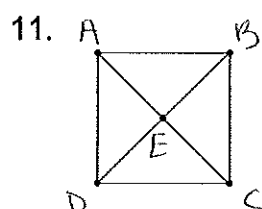
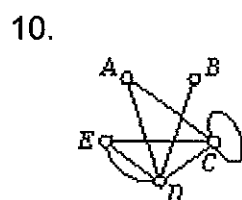
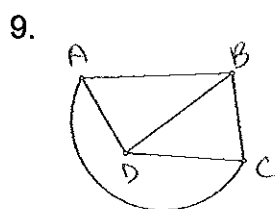
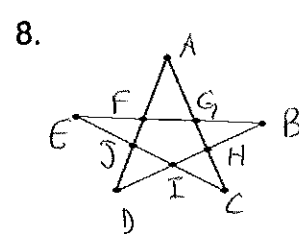
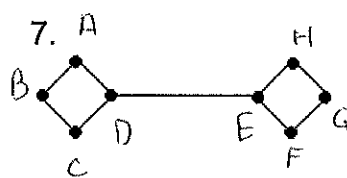
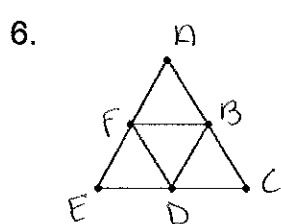
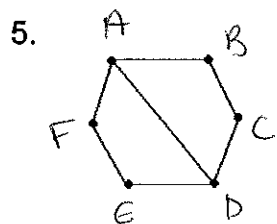


3.



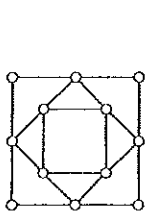
4.



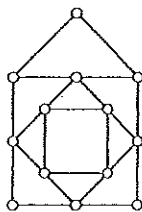


# Review of Euler Circuits and Paths. (4.4)

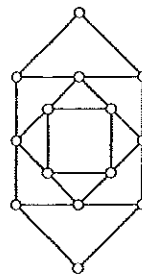
19. For each of the following, determine if the graph has an Euler circuit, an Euler path, or neither of these. Explain your answer but do not find the actual path or circuit.



(a)

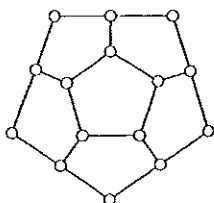


(b)

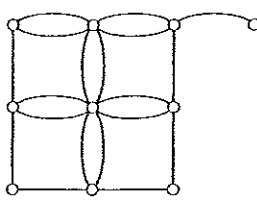


(c)

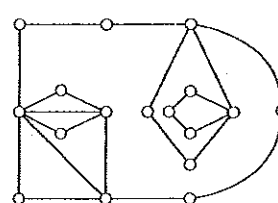
20. For each of the following, determine if the graph has an Euler circuit, an Euler path, or neither of these. Explain your answer but do not find the actual path or circuit.



(a)

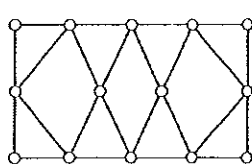


(b)

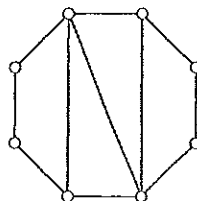


(c)

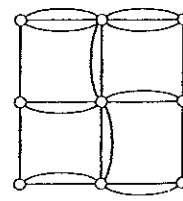
21. For each of the following, determine if the graph has an Euler circuit, an Euler path, or neither of these. Explain your answer but do not find the actual path or circuit.



(a)

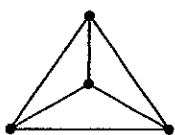


(b)

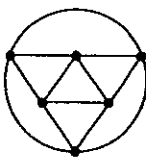


(c)

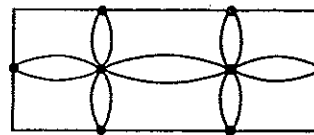
33. Which graphs in the figure below have Euler circuits? In the ones that do, find the Euler circuits by numbering the edges in the order the Euler circuit uses them. For the ones that don't, explain why no Euler circuit is possible.



(a)

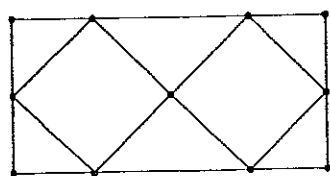


(b)

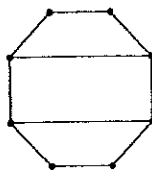


(c)

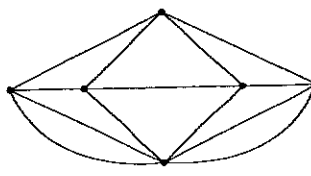
42. Each graph below represents the sidewalks to be cleaned in a fancy garden (one pass over a sidewalk will clean it). Can the cleaning be done using an Euler circuit? If so, show the circuit in each graph by numbering the edges in the order the Euler circuit uses them. If not, explain why no Euler circuit is possible.



(a)



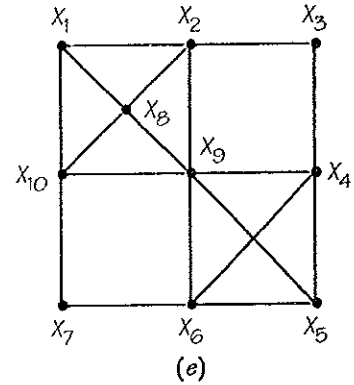
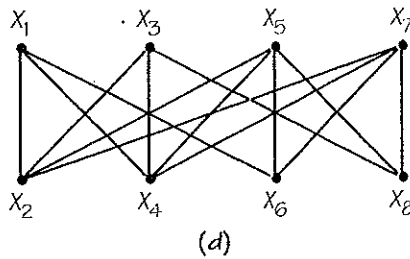
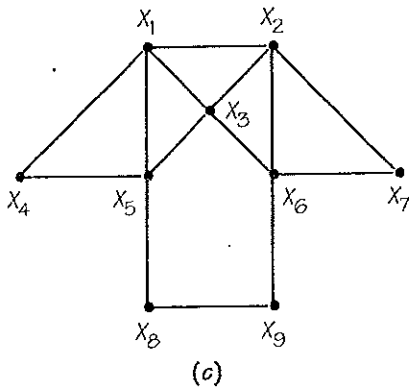
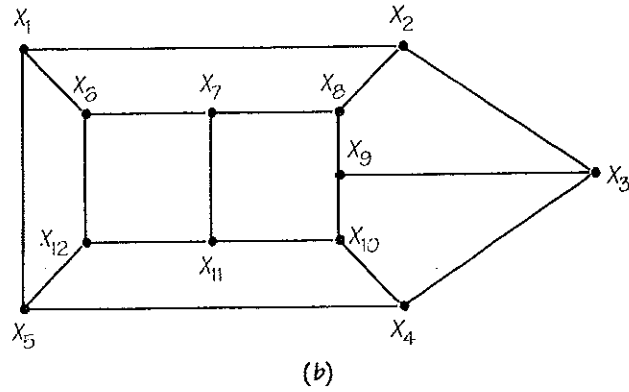
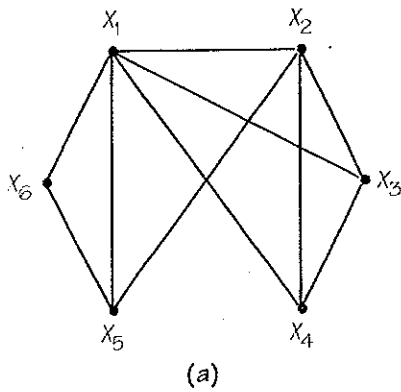
(b)



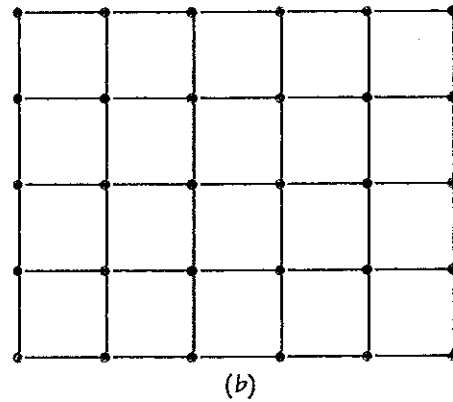
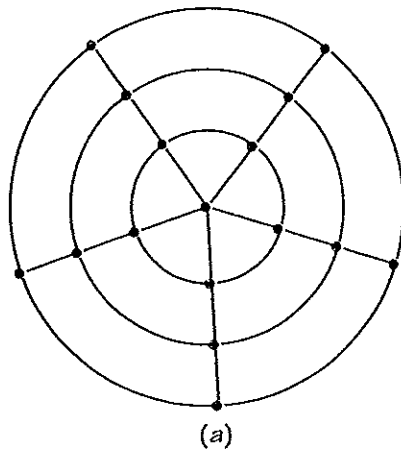
(c)

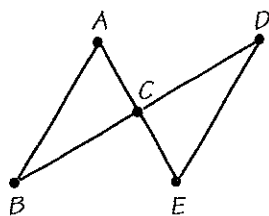
# Review of Hamiltonian Circuits and Paths. (4.5)

1. For each graph below, write a Hamiltonian circuit starting at  $X_1$ .



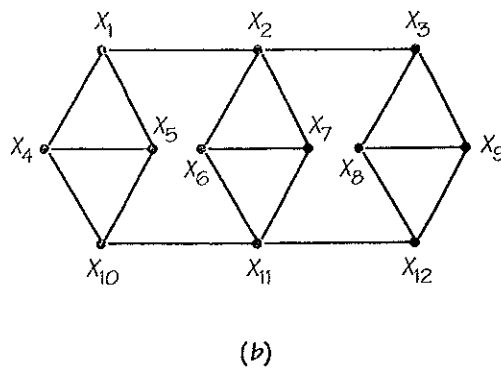
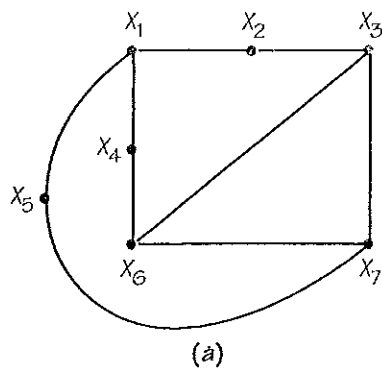
2. For each of the graphs below, add wiggly edges to indicate a Hamiltonian circuit.



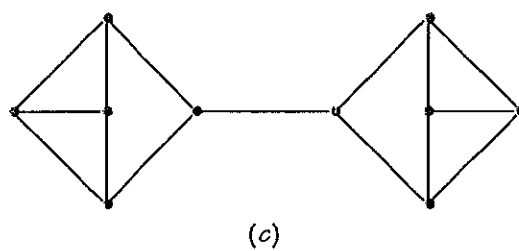
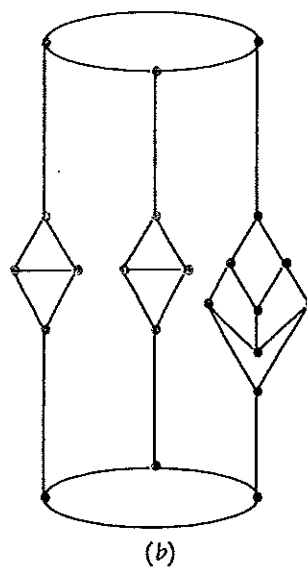
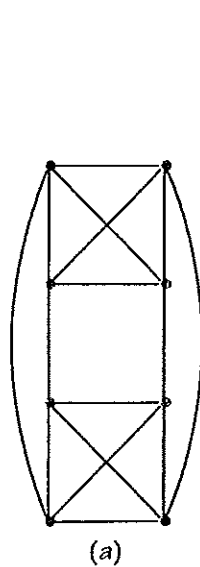


5. Explain why the tour  $ACEDCBA$  is not a Hamiltonian circuit for the graph at the left. Does this graph have a Hamiltonian circuit?

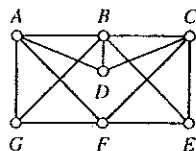
6. Do the following graphs have Hamiltonian circuits? If not, can you demonstrate why not?



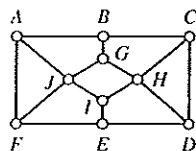
7. For each of the graphs below, determine if there is a Hamiltonian circuit.



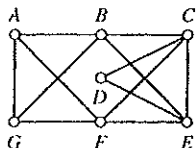
1. Two of the Hamilton circuits in the following graph are  $A, B, D, C, E, F, G, A$  and  $A, D, C, E, B, G, F, A$ . Find two more (not these circuits reversed). Use  $A$  as the starting point.



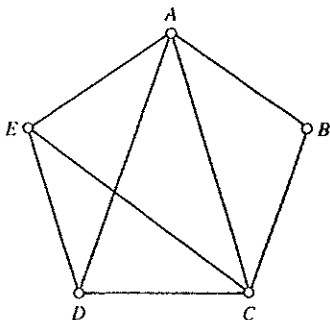
2. Find two Hamilton circuits in the following graph.



3. List all possible Hamilton circuits in the following graph.



4. List all possible Hamilton circuits in the following graph.  
*one*

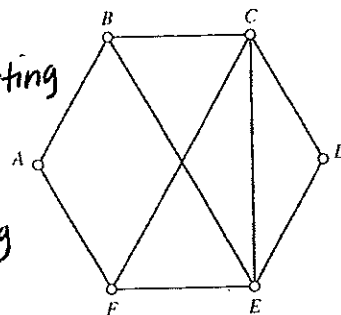


b. Is there an Euler Circuit or Path?  
if yes, name it.

5. List all possible Hamilton circuits in the following graph starting  
*one*  
(a) at vertex  $A$   
(b) at vertex  $D$ .

c. list an Euler Circuit starting at vertex  $A$ . (if possible)

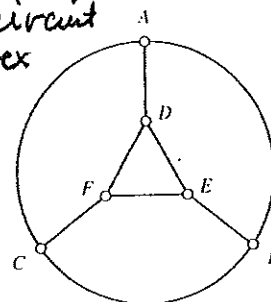
d. list an Euler path starting at vertex  $A$ . (if possible)



6. List all possible Hamilton circuits in the following graph starting  
*one*  
(a) at vertex  $A$   
(b) at vertex  $C$ .

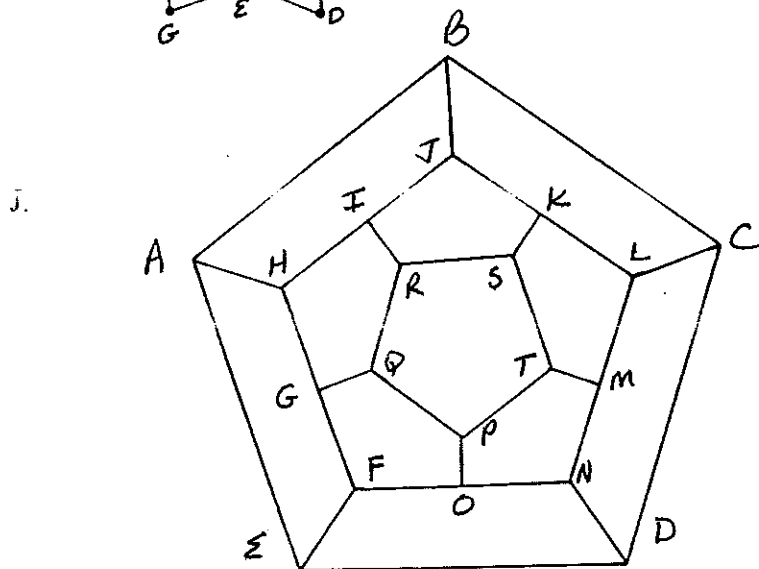
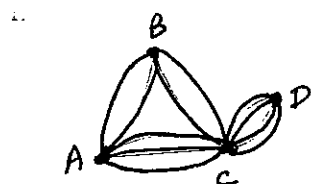
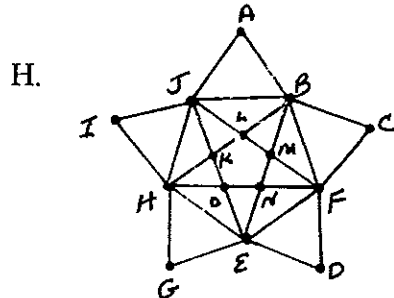
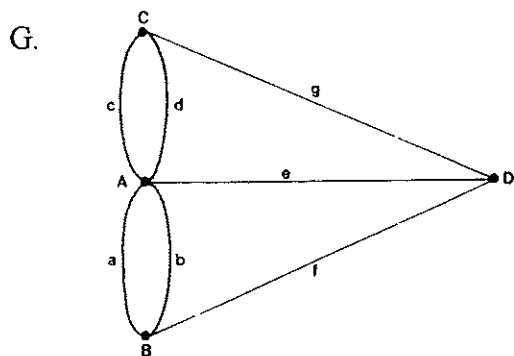
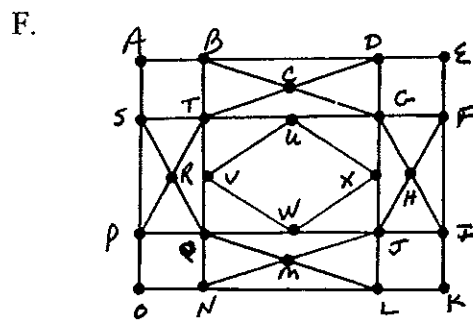
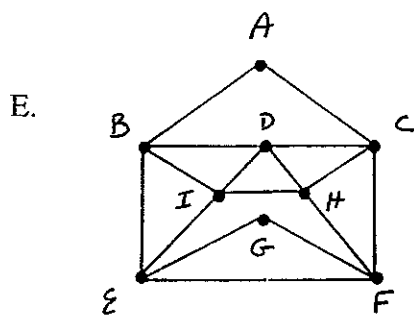
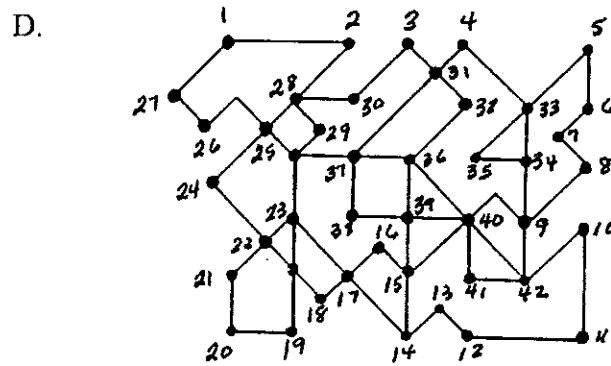
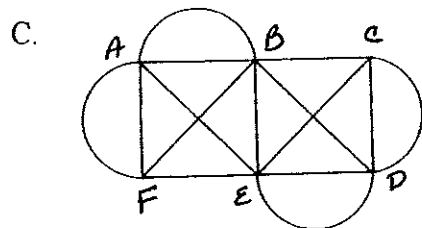
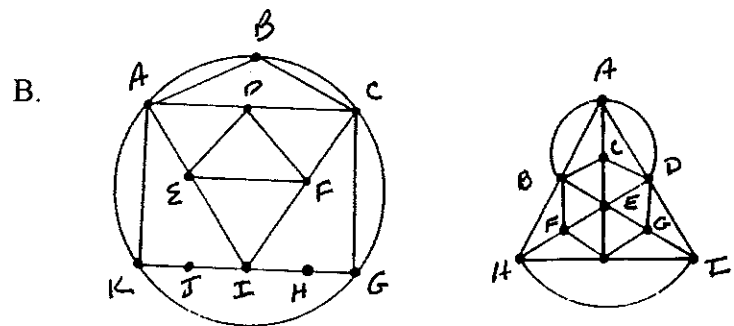
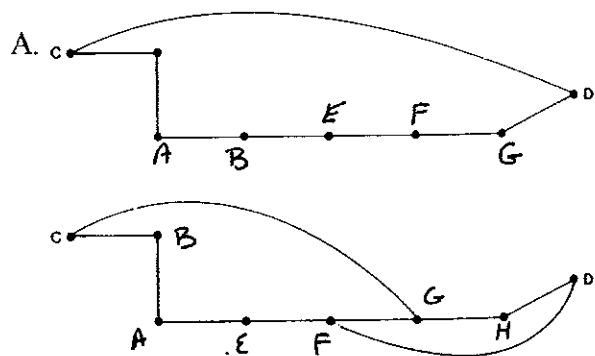
c. list an Euler circuit starting at vertex  $A$  (if possible).

d. list an Euler path starting at vertex  $A$  (if possible).





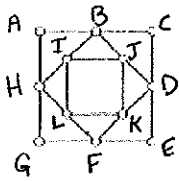
Examples:



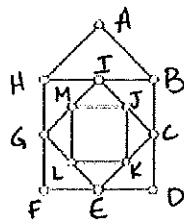
# Review of Euler Circuits and Paths and Hamiltonian Circuits and Paths

Do the following graphs have an Euler circuit? A path? If so, name it.

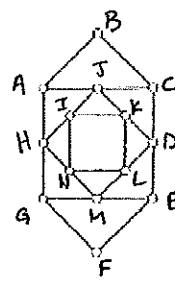
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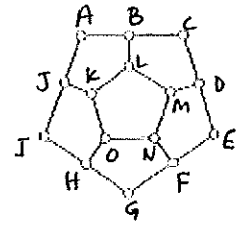
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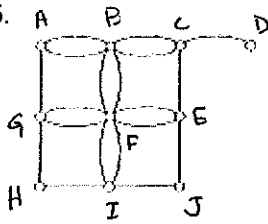
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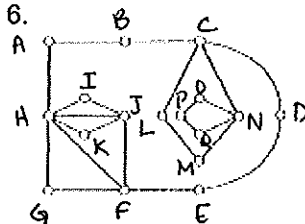
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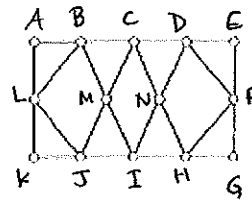
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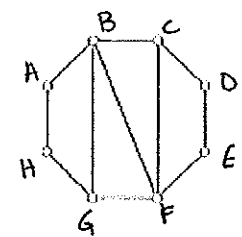
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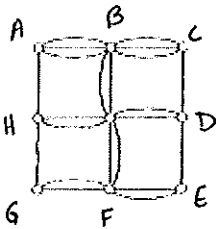
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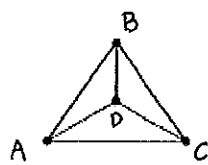
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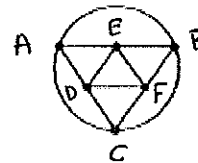
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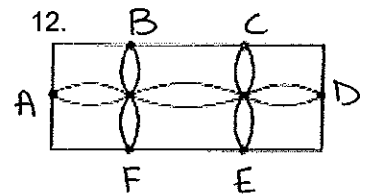
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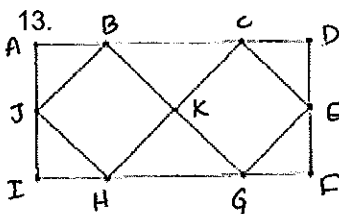
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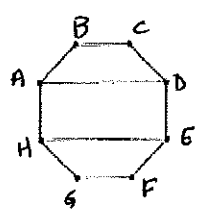
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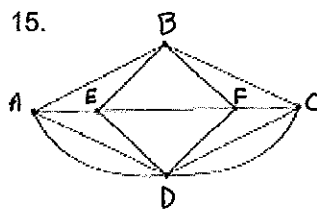
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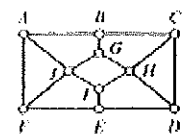
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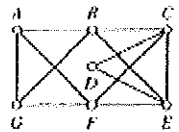
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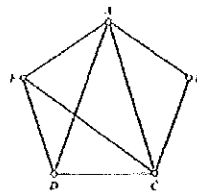
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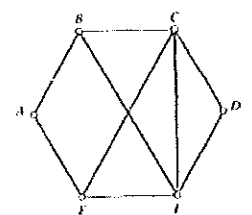
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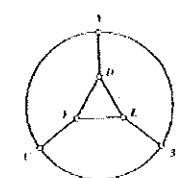
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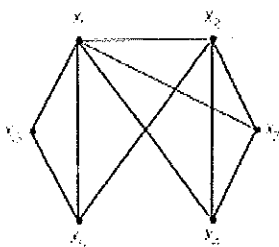
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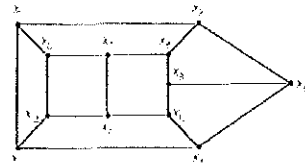
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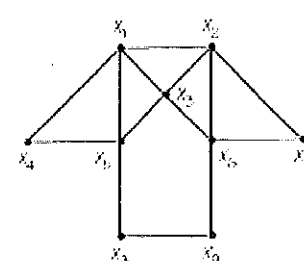
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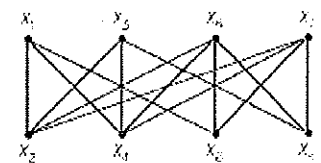
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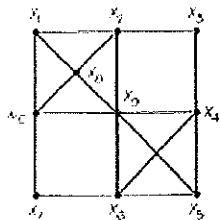
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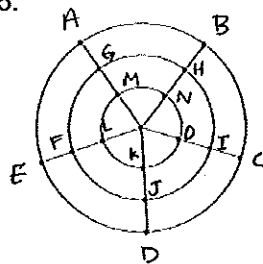
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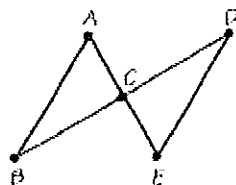
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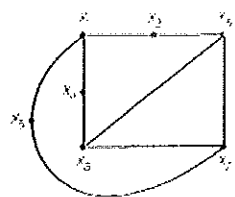
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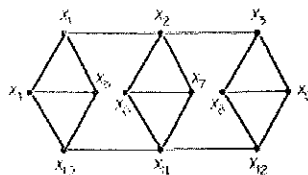
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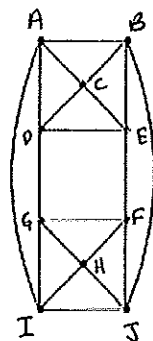
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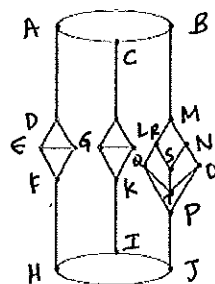
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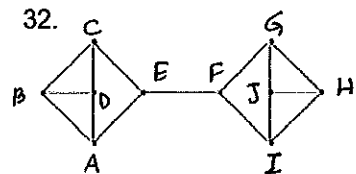
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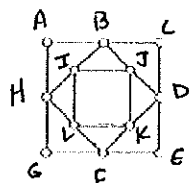


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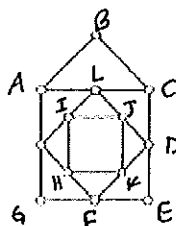


Do the following graphs have a Hamiltonian circuit? A path? If so, name it.

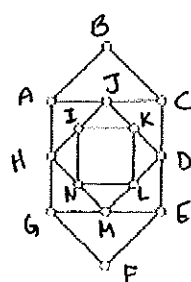
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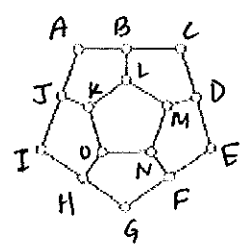
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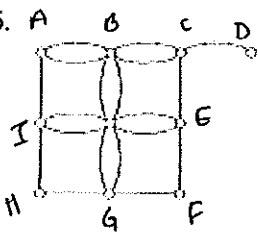
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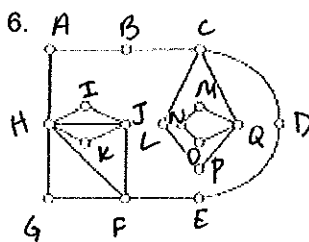
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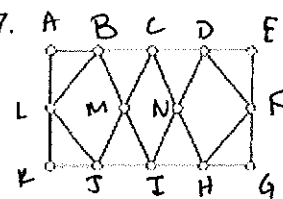
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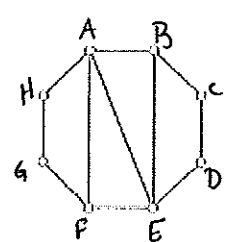
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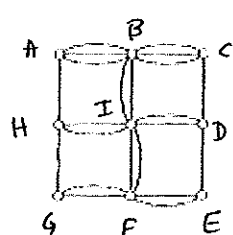
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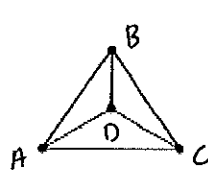
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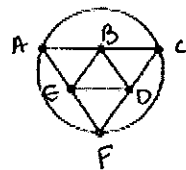
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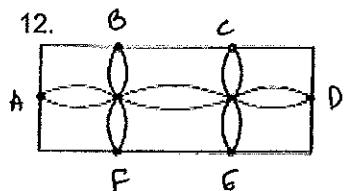
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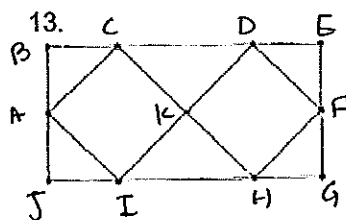
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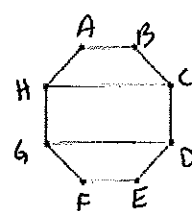
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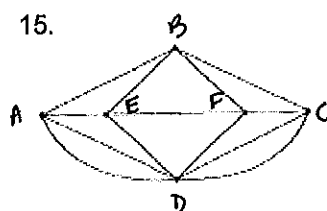
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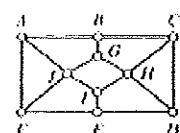
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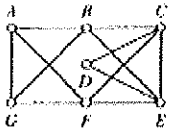
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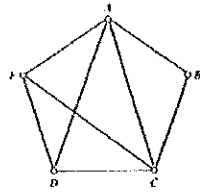
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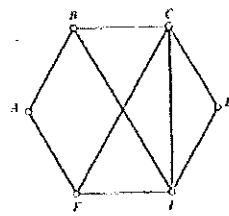
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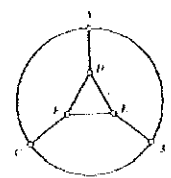
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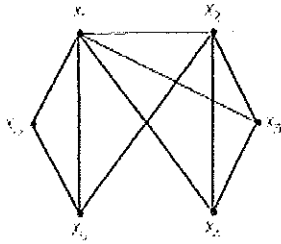
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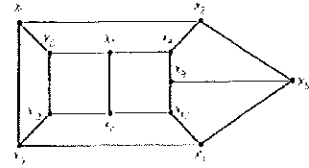
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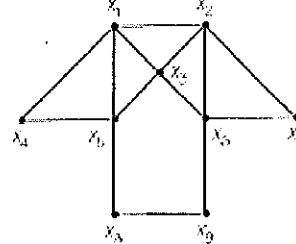
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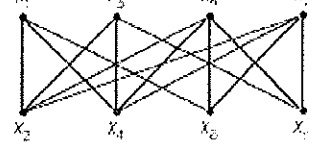
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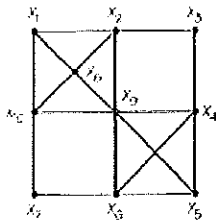
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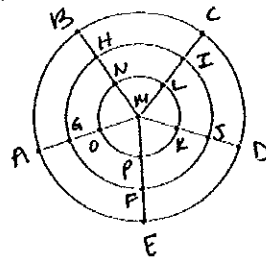
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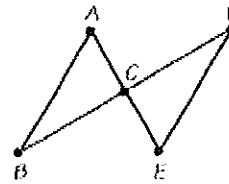
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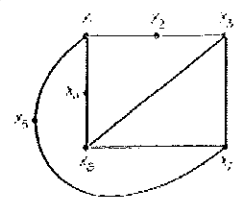
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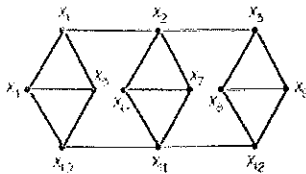
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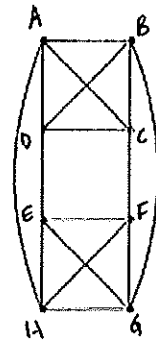
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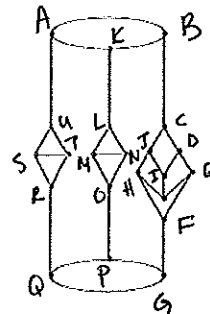
29.



30.



31.



32.

